TITLE:

Angehrn-Siu type effective freeness for quasi-log canonical pairs

AUTHOR(S):

Haidong, Liu

CITATION:

Haidong, Liu. Angehrn-Siu type effective freeness for quasi-log canonical pairs. 代数幾何学シンポジウム記録 2016, 2016: 146-146

ISSUE DATE:

2016

URL:

http://hdl.handle.net/2433/218282

RIGHT:

京都大学学術情報リポジトリ KURENAI 紅 Kyoto University Research Information Repository

Angehrn-Siu type effective freeness for quasi-log canonical pairs

Haidong Liu
Department of Mathematics, Graduate School of Science, Kyoto University

Background

All arguments are carried out over the complex number field $\mathbb{C}$. A *scheme* means a separated scheme of finite type.

Definition (Quasi log canonical singularities, cf. [1], [2])

A quasi-log scheme is a scheme X endowed with an $\mathbb{R}$-Cartier divisor (or $\mathbb{R}$-line bundle) ω on X, a proper closed subscheme $X_{-\infty} \subset X$, and a finite collection $\{C_i\}$ of reduced and irreducible subschemes of X such that there exists a proper morphism $f : (Y, B_Y) \to X$ from a globally embedded simple normal crossing pair satisfying the following properties:

- $K_Y + B_Y \sim_{\mathbb{R}} f^*\omega$.
- The natural map $\mathscr{O}_X \to f_*\mathscr{O}_Y(\lceil -(B_Y^{<1})\rceil)$ induces an isomorphism
$$\mathscr{I}_{X_{-\infty}} \cong f_*\mathscr{O}_Y(\lceil -(B_Y^{<1})\rceil - \lfloor B_Y^{>1}\rfloor),$$
where $\mathscr{I}_{X_{-\infty}}$ is the defining ideal sheaf of $X_{-\infty}$.
- The collection of subvarieties $\{C_i\}$ coincides with the images of (Y, B_Y)-strata that are not included in $X_{-\infty}$.

If $X_{-\infty} = \emptyset$, then we usually say that $[X, \omega]$ is a quasi-log canonical pair (a qlc pair, for short) or $[X, \omega]$ is a quasi-log scheme with only qlc singularities.

Known results of Angehrn–Siu type theorems

- Angehrn–Siu type theorems holds for adjoint bundles by [3] in 1995;
- Angehrn–Siu type theorems holds for klt singularities by [4] in 1997;
- Angehrn–Siu type theorems holds for lc singularities by [5] in 2010.

Motivation

Quasi-log canonical singularity

When we try to use induction on dimension in minimal model program, we can not avoid to deal with the non-normal varieties. Then the theory of mixed Hodge structures meets the MMP. Recently qlc singularities was introduced by Ambro and Fujino and used by Kollár and Kovács to prove that log canonical singularities are Du Bois for example. The qlc approach seems more and more powerful to deal with the non-normal case. An important result of qlc is the following theorem proved in this paper:

Theorem 1 (Inversion of adjunction)

Let $[X, \omega]$ be a qlc pair and B be an effective $\mathbb{R}$-Cartier divisor on X such that $SuppB$ contains no qlc centers of $[X, \omega]$. Let X' be a union of some qlc strata of $[X, \omega]$, then $[X, \omega + B]$ is qlc in a neighborhood of X' if and only if $[X', \omega|_{X'} + B|_{X'}]$ is qlc.

Semi-log canonical singularity

It is well known that when we consider the deformations of singularities and compactifications of moduli spaces, slc singularities are coming out and exactly at the boundary of moduli space of varieties with at worse log caonical singularities. So it is natural to ask as if Angehrn–Siu type theorems holds for slc too. Here we use the qlc technique to achieve this goal since we have the following result proved by Fujino, cf.[6]:

Known result
Let (X, Δ) be a quasi-projective semi log canonical pair. Then $[X, K_X + \Delta]$ has a quasi-log structure with only qlc singularities.

Main result

Theorem 2 (Effective base point freeness)

Let $[X, \omega]$ be a projective qlc pair such that ω is an $\mathbb{R}$-Cartier divisor and let M be a Cartier divisor on X such that $N = M - \omega$ is ample. Let $x \in X$ be a closed point. We assume that there are positive numbers $c(k)$ with the following properties:

- *If $x \in Z \subset X$ is an irreducible (positive dimensional) subvariety, then*
$$N^{\dim Z} \cdot Z > c(\dim Z)^{\dim Z}.$$
- *The numbers $c(k)$ satisfy the inequality:*
$$\sum_{k=1}^{\dim X} \frac{k}{c(k)} \leq 1.$$

Then $\mathcal{O}_X(M)$ has a global section not vanishing at x.

Theorem 3 (Effective point separation)

Let $[X, \omega]$ be a projective qlc pair such that ω is an $\mathbb{R}$-Cartier divisor and let M be a Cartier divisor on X such that $N = M - \omega$ is ample. Let $x_1, x_2 \in X$ be two closed points. We assume that there are positive numbers $c(k)$ with the following properties.

- *If $Z \subset X$ is an irreducible (positive dimensional) subvariety that contains x_1 or x_2, then*
$$N^{\dim Z} \cdot Z > c(\dim Z)^{\dim Z}.$$
- *The numbers $c(k)$ satisfy the inequality:*
$$\sum_{k=1}^{\dim X} 2^{1/k}\frac{k}{c(k)} \leq 1.$$

Then $\mathscr{O}_X(M)$ has a global section separating x_1 and x_2.

Remark.
The conclusions are also held for semi-log canonical singularities since slc singularities are qlc singularities by Fujino's result.

References.

[1] F. Ambro, Quasi-log varieties, Tr. Mat. Inst. Steklova **240** (2003), Biratsion. Geom. Linein. Sist. Konechno Porozhdennye Algebry, 220–239; translation in Proc. Steklov Inst. Math. 2003, no. 1 (240), 214–233.

[2] O. Fujino, Foundation of the minimal model program, preprint (2014).

[3] U. Angehrn, Y.-T. Siu, Effective freeness and point separation for adjoint bundles, Invent. Math. **122** (1995), no. 2, 291–308.

[4] J. Kollár, Singularities of pairs, *Algebraic geometry—Santa Cruz 1995*, 221–287, Proc. Sympos. Pure Math., **62**, Part 1, Amer. Math. Soc., Providence, RI, 1997.

[5] O. Fujino, Effective base point free theorem for log canonical pairs, II. Angehrn–Siu type theorems, Michigan Math. J. **59** (2010), no. 2, 303–312.

[6] O. Fujino, Fundamental theorems for semi log canonical pairs, Algebr. Geom. **1** (2014), no. 2, 194–228.